LaTeX Mathematics Examples

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June 1, 2020

Contents

1 Theorems et al.

Definition 1 (right-angled triangles)**.** *A* right-angled triangle *is a triangle whose sides of length a, b and c, in some permutation of order, satisfies $a^2 + b^2 = c^2$.*

Lemma 2. *The triangle with sides of length 3, 4 and 5 is right-angled.*

This lemma follows from Definition 1 since $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

Theorem 3 (Pythagorean triplets)**.** *Triangles with sides of length $a = p^2 - q^2$, $b = 2pq$ and $c = p^2 + q^2$ are right-angled triangles.*

Prove this Theorem 3 by the algebra $a^2 + b^2 = (p^2 - q^2)^2 + (2pq)^2 = p^4 - 2p^2q^2 + q^4 + 4p^2q^2 = p^4 + 2p^2q^2 + q^4 = (p^2 + q^2)^2 = c^2$.

2 Accents

Mathematical accents are performed by a short command with one argument, such as

$$\tilde{f}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} \, dx \,,$$

or

$$\dot{\vec{\omega}} = \vec{r} \times \vec{I} \,.$$

3 Relations

LaTeX knows to typeset extra space around relations $=$, $\approx$, and

- inequalities $<, >, \leq, \geq$

- very much so $\ll, \gg$
- set relations $\in, \subset$
- and so on.

For example, $0 < \epsilon \ll 1$ is `0<\epsilon\ll1`, not the common error `0<\epsilon<<1` which produces the ugly $0 < \epsilon << 1$.

4 Delimiters

See how the delimiters are of reasonable size in these examples

$$(a+b)\left[1-\frac{b}{a+b}\right]=a\,,$$

$$\sqrt{|xy|}\leq\left|\frac{x+y}{2}\right|,$$

even when there is no matching delimiter

$$\int_a^b u\frac{d^2v}{dx^2}\,dx=\left.u\frac{dv}{dx}\right|_a^b-\int_a^b\frac{du}{dx}\frac{dv}{dx}\,dx.$$

5 Spacing

Differentials often need a bit of help with their spacing as in

$$\iint xy^2\,dx\,dy=\frac{1}{6}x^2y^3,$$

whereas vector problems often lead to statements such as

$$u=\frac{-y}{x^2+y^2}\,,\quad v=\frac{x}{x^2+y^2}\,,\quad\text{and}\quad w=0\,.$$

Occasionally one gets horrible line breaks when using a list in mathematics such as listing the first twelve primes $2, 3, 5, 7, 11, 13, 17, 19, 23$
In such cases, perhaps include `\mathcode‘\,="213B` inside the inline maths environment so that the list breaks: $2 \,,\, 3 \,,\, 5 \,,\, 7 \,,\, 11 \,,\, 13 \,,\, 17 \,,\, 19 \,,\, 23 \,,\, 29 \,,\, 31 \,,\, 37$. Be discerning about when to do this as the spacing is different.

6 Functions

Observe that trigonometric and other elementary functions are typeset properly, even to the extent of providing a thin space if followed by a single letter argument:

$$\exp(i\theta) = \cos\theta + i\sin\theta\,, \quad \sinh(\log x) = \frac{1}{2}\left(x - \frac{1}{x}\right).$$

With sub- and super-scripts placed properly on more complicated functions,

$$\lim_{q\to\infty} \|f(x)\|_q = \max_x |f(x)|,$$

and large operators, such as integrals and

$$\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{where } n! = \prod_{i=1}^{n} i\,, \\ \overline{U_\alpha} &= \bigcap_\alpha U_\alpha\,. \end{aligned}$$

In inline mathematics the scripts are correctly placed to the side in order to conserve vertical space, as in $1/(1-x) = \sum_{n=0}^{\infty} x^n$.

7 Command definition

The Airy function, $\mathrm{Ai}(x)$, may be incorrectly defined as this integral

$$\mathrm{Ai}(x) = \int \exp(s^3 + isx)\, ds\,.$$

This vector identity serves nicely to illustrate two of the new commands:

$$\boldsymbol{\nabla} \times \boldsymbol{q} = \boldsymbol{i}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + \boldsymbol{j}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \boldsymbol{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right).$$

8 Arrays

Arrays of mathematics are typeset using one of the matrix environments as in

$$\begin{bmatrix} 1 & x & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 + xy \\ y - 1 \end{bmatrix}.$$

Case statements use cases:

$$|x| = \begin{cases} x, & \text{if } x \geq 0\,, \\ -x, & \text{if } x < 0\,. \end{cases}$$

Many arrays have lots of dots all over the place as in

$$\begin{matrix} -2 & 1 & 0 & 0 & \cdots & 0 \\ 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 \\ 0 & 0 & 1 & -2 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & 1 \\ 0 & 0 & 0 & \cdots & 1 & -2 \end{matrix}$$

9 Equation arrays

In the flow of a fluid film we may report

$$u_\alpha = \epsilon^2 \kappa_{xxx} \left(y - \frac{1}{2} y^2 \right), \tag{1}$$

$$v = \epsilon^3 \kappa_{xxx} y \,, \tag{2}$$

$$p = \epsilon \kappa_{xx} \,. \tag{3}$$

Alternatively, the curl of a vector field (u, v, w) may be written with only one equation number:

$$\begin{aligned} \omega_1 &= \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \,, \\ \omega_2 &= \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \,, \\ \omega_3 &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \,. \end{aligned} \tag{4}$$

Whereas a derivation may look like

$$\begin{aligned} (p \wedge q) \vee (p \wedge \neg q) &= p \wedge (q \vee \neg q) \quad \text{by distributive law} \\ &= p \wedge T \quad \text{by excluded middle} \\ &= p \quad \text{by identity} \end{aligned}$$

Use subequations for parts of a whole The `subequations` environment generates labels for the enclosed mathematics which are a base number followed by a,b,c,... in sequence, as illustrated below.

$$u_\alpha = \epsilon^2 \kappa_{xxx} \left(y - \frac{1}{2} y^2 \right), \tag{5a}$$

$$v = \epsilon^3 \kappa_{xxx} y \,, \tag{5b}$$

$$p = \epsilon \kappa_{xx} \,. \tag{5c}$$

Cross-reference to any individual equation or to the collective (5).

Typesetting long expressions Recall that typesetting multi-line mathematics is an art normally too hard for computer recipes. Nonetheless, if you need to be automatically flexible about multi-line long algebraic expressions, and you do not mind some crude typesetting, then perhaps invoke `\parbox` to help as follows:

$$u_1 = -2\gamma\epsilon^2 s_2 + \mu\epsilon^3\big(\tfrac{3}{8}s_2 + \tfrac{1}{8}s_1 i\big) + \epsilon^3\big(- \tfrac{81}{32}s_4 s_2^2 - \tfrac{27}{16}s_4 s_2 s_1 i + \tfrac{9}{32}s_4 s_1^2 + \tfrac{27}{32}s_3 s_2^2 i - \tfrac{9}{16}s_3 s_2 s_1 - \tfrac{3}{32}s_3 s_1^2 i\big) + \int_a^b 1 - 2x + 3x^2 - 4x^3\, dx$$

The `breqn` package is not yet reliable enough for general use.

Also, remember to use `\parbox` to typeset multiline entries in tables.