# A normal form of your dynamical system

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Generally, the lowest order, most important, terms are near the end of each expression.

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```
xrhs:={};
yrhs:={-y(1)+re*y(2)-
small*(y(1)^2+y(2)^2)*y(2)+w(1)
,-2*y(2)-small*(y(1)^2+y(2)^2)*y(1)+w(1) };
zrhs:={ };
toosmall:=6;
theman:=csuman;
factor sig,small,re;
```

### Specified dynamical system

$$\dot y_1 = \mathit{re}\varepsilon y_2 + \sigma w_1 + \varepsilon^2( - y_2^3 - y_2y_1^2) - y_1$$

$$\dot y_2 = \sigma w_1 + \varepsilon^2( - y_2^2y_1 - y_1^3) - 2y_2$$

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### Time dependent normal form coordinates

$$y_1 = \mathit{re}^2\sigma\varepsilon^4( - 1/4e^{3t}\star w_1\, Y_2^2 + 5/12e^{2t}\star w_1\, Y_2^2 - 2e^{2t}\star w_1\, Y_2Y_1 + 3e^{t}\star w_1\, Y_2Y_1 + 1/2e^{t}\star w_1\, Y_1^2 - 7/12e^{-1t}\star w_1\, Y_2^2 + e^{-1t}\star w_1\, Y_2Y_1 - 3/2e^{-1t}\star w_1\, Y_1^2 + 3/4e^{-2t}\star w_1\, Y_2^2 + 1/2e^{-2t}\star w_1\, Y_1^2) + \mathit{re}^2\varepsilon^4(1/4Y_2^3 + 1/2Y_2Y_1^2) + \mathit{re}\sigma\varepsilon^3( - 1/4e^{3t}\star w_1\, Y_2^2 - 1/3e^{2t}\star w_1\, Y_2^2 + 1/6e^{2t}\star w_1\, Y_2Y_1 - 2/3e^{t}\star w_1\, Y_2Y_1 + 3/2e^{t}\star w_1\, Y_1^2 - 7/12e^{-1t}\star w_1\, Y_2^2 + 2/3e^{-1t}\star w_1\, Y_2Y_1 - 3/2e^{-1t}\star w_1\, Y_1^2 - 7/6e^{-2t}\star w_1\, Y_2Y_1) + \mathit{re}\sigma\varepsilon(e^{-1t}\star w_1 - e^{-2t}\star w_1) + \mathit{re}\varepsilon^3( - 7/12Y_2^2Y_1 - 1/2Y_1^3) - \mathit{re}\varepsilon Y_2 + \sigma\varepsilon^4( - 1/40e^{7t}\star w_1\, Y_2^4 - 1/10e^{6t}\star w_1\, Y_2^3Y_1 - 16/45e^{5t}\star w_1\, Y_2^2Y_1^2 + 8/9e^{4t}\star w_1\, Y_2Y_1^3 + 2/5e^{3t}\star w_1\, Y_2^3Y_1 + 1/4e^{3t}\star w_1\, Y_1^4 + 1/5e^{2t}\star w_1\, Y_2^4 + 5/9e^{2t}\star w_1\, Y_2^2Y_1^2 + 2/5e^{t}\star w_1\, Y_2^3Y_1 + 4/9e^{t}\star w_1\, Y_2Y_1^3 +$$

$7/40\mathrm{e}^{-1t}\star w_1\, Y_2^4 + 2\mathrm{e}^{-1t}\star w_1\, Y_2^2 Y_1^2 + 5/4\mathrm{e}^{-1t}\star w_1\, Y_1^4 + 7/10\mathrm{e}^{-2t}\star w_1\, Y_2^3 Y_1 + 4/3\mathrm{e}^{-2t}\star w_1\, Y_2 Y_1^3) + \sigma\varepsilon^2(3/5\mathrm{e}^{3t}\star w_1\, Y_2^2 + 2/3\mathrm{e}^{2t}\star w_1\, Y_2 Y_1 + 1/3\mathrm{e}^{t}\star w_1\, Y_1^2 + 2/3\mathrm{e}^{-1t}\star w_1\, Y_2 Y_1 + 3/5\mathrm{e}^{-2t}\star w_1\, Y_2^2 + 1/3\mathrm{e}^{-2t}\star w_1\, Y_1^2) + \sigma\mathrm{e}^{-1t}\star w_1\, + \varepsilon^4(7/40 Y_2^4 Y_1 + 2/3 Y_2^2 Y_1^3 + 1/4 Y_1^5) + \varepsilon^2(1/5 Y_2^3 + 1/3 Y_2 Y_1^2) + Y_1$

$y_2 = re^2\sigma\varepsilon^4(\mathrm{e}^{2t}\star w_1\, Y_2^2 - \mathrm{e}^{t}\star w_1\, Y_2 Y_1 + \mathrm{e}^{-1t}\star w_1\, Y_2^2 - 3\mathrm{e}^{-1t}\star w_1\, Y_2 Y_1 + 2\mathrm{e}^{-2t}\star w_1\, Y_2 Y_1) + re^2\varepsilon^4 Y_2^2 Y_1 + re\sigma\varepsilon^3( - 5/12\mathrm{e}^{2t}\star w_1\, Y_2^2 - 3\mathrm{e}^{t}\star w_1\, Y_2 Y_1 + 1/3\mathrm{e}^{-1t}\star w_1\, Y_2^2 - 3\mathrm{e}^{-1t}\star w_1\, Y_2 Y_1 + 3\mathrm{e}^{-1t}\star w_1\, Y_1^2 - 3/4\mathrm{e}^{-2t}\star w_1\, Y_2^2 - 3/2\mathrm{e}^{-2t}\star w_1\, Y_1^2) + re\varepsilon^3( - 1/4 Y_2^3 - 3/2 Y_2 Y_1^2) + \sigma\varepsilon^4( - 3/40\mathrm{e}^{6t}\star w_1\, Y_2^4 + 4/45\mathrm{e}^{5t}\star w_1\, Y_2^3 Y_1 - 14/9\mathrm{e}^{4t}\star w_1\, Y_2^2 Y_1^2 + 1/5\mathrm{e}^{3t}\star w_1\, Y_2^4 + 9/5\mathrm{e}^{3t}\star w_1\, Y_2^2 Y_1^2 - \mathrm{e}^{3t}\star w_1\, Y_2 Y_1^3 + 4/9\mathrm{e}^{2t}\star w_1\, Y_2^3 Y_1 + 2\mathrm{e}^{2t}\star w_1\, Y_2 Y_1^3 - 1/4\mathrm{e}^{2t}\star w_1\, Y_1^4 + 5/9\mathrm{e}^{t}\star w_1\, Y_2^2 Y_1^2 + \mathrm{e}^{t}\star w_1\, Y_1^4 + 8/15\mathrm{e}^{-1t}\star w_1\, Y_2^3 Y_1 + 3\mathrm{e}^{-1t}\star w_1\, Y_2 Y_1^3 + 1/8\mathrm{e}^{-2t}\star w_1\, Y_2^4 + 4/5\mathrm{e}^{-2t}\star w_1\, Y_2^2 Y_1^2 + 3/4\mathrm{e}^{-2t}\star w_1\, Y_1^4) + \sigma\varepsilon^2(1/3\mathrm{e}^{2t}\star w_1\, Y_2^2 + 2/3\mathrm{e}^{t}\star w_1\, Y_2 Y_1 + 1/3\mathrm{e}^{-1t}\star w_1\, Y_2^2 + 3\mathrm{e}^{-1t}\star w_1\, Y_1^2 + 2/3\mathrm{e}^{-2t}\star w_1\, Y_2 Y_1) + \sigma\mathrm{e}^{-2t}\star w_1\, + \varepsilon^4(1/40 Y_2^5 + 4/15 Y_2^3 Y_1^2 + 3/4 Y_2 Y_1^4) + \varepsilon^2(1/3 Y_2^2 Y_1 + Y_1^3) + Y_2$

**Result normal form DEs**

$\dot{Y}_1 = re^3\sigma^2\varepsilon^5( - \mathrm{e}^{-1t}\star w_1\, w_1 Y_1 + 1/2\mathrm{e}^{-2t}\star w_1\, w_1 Y_1) + re^2\sigma^2\varepsilon^4( - 4\mathrm{e}^{-1t}\star w_1\, w_1 Y_1 + 2\mathrm{e}^{-2t}\star w_1\, w_1 Y_1) + re\sigma^2\varepsilon^3( - 11/3\mathrm{e}^{-1t}\star w_1\, w_1 Y_1 - 1/6\mathrm{e}^{-2t}\star w_1\, w_1 Y_1) + \sigma^2\varepsilon^2( - 2/3\mathrm{e}^{-1t}\star w_1\, w_1 Y_1 - 2/3\mathrm{e}^{-2t}\star w_1\, w_1 Y_1) - Y_1$

$\dot{Y}_2 = re^3\sigma^2\varepsilon^5(\mathrm{e}^{-1t}\star w_1\, w_1 Y_2 - 1/2\mathrm{e}^{-2t}\star w_1\, w_1 Y_2) + re^2\sigma^2\varepsilon^4(4\mathrm{e}^{-1t}\star w_1\, w_1 Y_2 - 2\mathrm{e}^{-2t}\star w_1\, w_1 Y_2) + re\sigma^2\varepsilon^3(7/3\mathrm{e}^{-1t}\star w_1\, w_1 Y_2 + 5/6\mathrm{e}^{-2t}\star w_1\, w_1 Y_2) - 3/2 re\sigma\varepsilon^3 w_1 Y_1^2 + \sigma^2\varepsilon^2( - 2/3\mathrm{e}^{-1t}\star w_1\, w_1 Y_2 - 2/3\mathrm{e}^{-2t}\star w_1\, w_1 Y_2) - 3\sigma\varepsilon^2 w_1 Y_1^2 - 2Y_2$