A normal form of your dynamical system

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10:17am, May 30, 2023

Generally, the lowest order, most important, terms are near the end of each expression.

Specified dynamical system

$\dot{x}_1 = -\frac{\mathrm{d}\,x_1}{\mathrm{d}\,t} z_1\varepsilon - x_1 y_1\varepsilon$

$\dot{y}_1 = -2\frac{\mathrm{d}\,y_1}{\mathrm{d}\,t} y_1\varepsilon + 2x_1^2 y_1\varepsilon + x_1^2\varepsilon - y_1$

$\dot{z}_1 = -3\frac{\mathrm{d}\,z_1}{\mathrm{d}\,t} x_1\varepsilon + 2z_1$

off echo;

Time dependent normal form coordinates

$z_1 = O(\varepsilon^3,\sigma^2) + 6X_1Y_1Z_1\varepsilon^2 + Z_1$

$y_1 = O(\varepsilon^3,\sigma^2) + 2X_1^4\varepsilon^2 - 4X_1^2Y_1^2\varepsilon^2 + X_1^2\varepsilon + 6Y_1^3\varepsilon^2 - 2Y_1^2\varepsilon + Y_1$

$x_1 = O(\varepsilon^3,\sigma^2) + 2X_1^3Y_1\varepsilon^2 - 1/2X_1Y_1^2\varepsilon^2 + X_1Y_1Z_1\varepsilon^2 + X_1Y_1\varepsilon + X_1$

Result normal form DEs

$\dot{Z}_1 = O(\varepsilon^4,\sigma^3) - 54X_1^3Z_1\varepsilon^3 + 18X_1^2Z_1\varepsilon^2 - 6X_1Z_1\varepsilon + 2Z_1$

$\dot{Y}_1 = O(\varepsilon^4,\sigma^3) + 8X_1^4Y_1\varepsilon^3 + 4X_1^2Y_1\varepsilon^2 + 2X_1^2Y_1\varepsilon - Y_1$

$\dot{X}_1 = O(\varepsilon^4,\sigma^3) - 2X_1^5\varepsilon^3 - X_1^3\varepsilon^2 - 2X_1Y_1^2Z_1\varepsilon^3$